Analytic and semi-analytic solutions to classical color glass models

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The initial phase of a collision of two nuclei at very high energy is thought to be described by the dynamcis of color glass condensate (CGC) [1,2]. Color glass condensate refers to the fact that the wave function of highly boosted nuclei is expected to be dominated by gluons with transverse polarization and at occupation numbers large enough to enable classical field concepts to apply. The first viable model of CGC became known as the McLerran-Venugopalan (MV) model [2]. Nuclear collisions can be computed as collisions of sheets of CGC, leading to a state usually called glasma. The output of such calculations can then be used to initialize calculations of the subsequent stages of a nuclear collision, in particular the approach to kinetic equilibrium and the following near-equilibrium fluid dynamic evolution of the system.

The IP-glasma model [3] offers a numerical implementation of the MV model which has been successfully used over the past decade in a variety of phenomenological calculations. Analytic solutions to the MV-model have been offered in [1] based on a series expansion in proper time. However, the convergence of the series is poor and only the earliest features, like the onset of radial flow in the glasma, can be studied analytically.

In the current project we have computed the energy momentum tensor of the glasma in the MV model analytically for all times in the weak field limit. Weak-field limit here means that all non-abelian terms after the initial non-abelian interaction of the sheets of color glass are dropped from the calculation. Technically this means that the fields we consider are of order g^3A^2 where g is the coupling constant of the strong force, and A generically denotes a gauge potential in one of the colliding nuclei. The restriction to weak fields allows for the analytic solution to be obtained. Collisions of lead nuclei at the Large Hadron Collider are strictly speaking outside of the applicability of the weak field limit. Thus, while valuable insights can be obtained from analytic solutions, their applicability to real collision system will have to be carefully investigated.

We find that the energy momentum tensor in the simplest case of nuclei of homogeneous average color charge densities $\langle\rho^2\rangle \sim \mu_1, \mu_2$ in nucleus 1 and 2, resp., is given by Meijer-G functions. For example, the energy density from longitudinal fields is [4]

$$\langle E_z^2 + B_z^2\rangle = \frac{2\pi\alpha_s^3 N_c}{N_c^2 - 1}\mu_1\mu_2\frac{M(4m^2\tau^2)}{\tau}$$

where $N_c = 3$ is the number of colors, $\alpha_s = g^2/4\pi$, and M is a linear combination of three Meijer-G functions. The averaging here is over all color charge densities that are allowed for given μ_1, μ_2. The McLerran-Venugopalan model suffers from infrared (IR) and ultraviolet (UV) divergences which require ad-hoc regularizations. The former is indicated by the presence of an IR cutoff m in the result above, which is chosen to be implemented as an effective gluon mass in the Green's function for the gluon field. Fig. 1 shows a comparison of our results for the reduced longitudinal and transverse pressure with a numerical calculation using the publically available IP glasma code. The two calculations agree over most of the time but differ for times from around 0.1 to 0.4 fm. Some disagreement is to be expected as scales would have to be carefully matched between the discretized numerical solution and our analytic solution. However, agreement is reached in the long time limit.

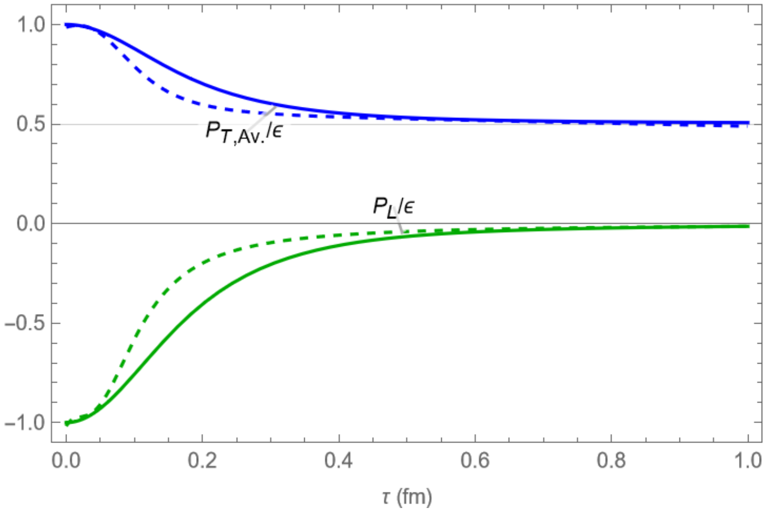

FIG. 1. Ratios of transverse pressure $\boldsymbol{p_T}$ and longitudinal pressure $\boldsymbol{p_L}$ over the energy density $\boldsymbol{\epsilon}$ as functions of proper time $\boldsymbol{\tau}$ in the McLerran-Venugopalan model [4]. Solid lines: analytic result. Dashed line: numerical solution using the IP glasma code [3].

The divergences in the MV-model have motivated us to propose a new model which is called the improved Gaussian model. It removes UV-divergences by smearing the initial charge distribution on small but finite length scales. It also implements global color neutrality which considerably softens (but not completely removes) the IR divergences. The improved Gaussian model is more challenging to handle in analytic calculations, but we have computed expressions for the energy momentum tensor of the glasma up to one integral which remains to be done numerically [4].

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[1] G. Chen, R. J. Fries, J. I. Kapusta, Y. Li, Phys. Rev. C **92**, 064912 (2015).
[2] L. D. McLerran, R. Venugopalan, Phys. Rev. D **49**, 2233 (1994).
[3] B. Schenke, P. Tribedy , R. Venugopalan, Phys. Rev. Lett. **108**, 252301 (2012).
[4] S. Robicheaux and R. J. Fries, (2024) submitted.